Dynamics of Cumulative Culture with Microfoundation

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1. Introduction

Theories on cumulative culture have been developed mainly in the field of cultural evolution which is an application of darwinian evolutinary process to cultural phenomena. To find the reason why only human-beings appear to achieve high cultural complexity, they have not simply quoted the concepts of preceding studies represented by Boyd and Richerson (1985) but focused on several aspects which might be related to the accumulation of cultural traits. For example, Henrich(2004) presented the contribution of population size to cumulative culture; Mesoudi(2011a) clearfied the restriction caused by the acquistion cost of accumulated knowledge; Lehman et al.(2013) calculated the optimal strategy of time allocation for learning schedules.

The aim of this study is to add a different dynamics by using the methodology of economic growth theory which values rationality and expectation of individuals. In general, economics in the mainstream has been considered to be different from cultural evolution in terms of both concepts and methods[i]. However, if we focus on the cumulative aspects, there are actually several similarities including the following two points. First, both deal with the macro-scale dynamics resulting from the accumulation of some micro-scale activities. Economics has also provided various microfounded dynamic models for capital accumulation, whereas static aspects are often emphasized by cultural evolutionists. Second, both have a strong tendency of prediction or purpose-orientation. Although the tendency has been treated carefully in cultural evolution as "guided variation", cumulative culture is especially the field where it contains since cultural accumulation is almost peculiar to human-beings who seem to be more rational than other species. Therefore, we can say that it is worth applying the methodology of economic growth theory to cumulative cultural evolution.

This paper is composed of five chapters including this introduction. Chapter 2 provides an explanation of the structure of our model. Chapter 3 deals with the derivation of its

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steady-state with several interpretations. Then, Chapter 4 covers the confirmation of its stability by means of phase diagrams. Finally, Chapter 5 summarizes implication and conclusion.

2. Model

To emphasize the role of rationality and expectation, our model is constructed of two main components: cultural stock and individuals seeking to maxmize their utility.

Cultural stock is the state variable which denotes the amount of accumulated knowledge in a society[ii]. Note that it is assumed to be homogeneous for simplification; in other words, the effect of cultural diversity is eliminated here. Individuals, on the other hand, have the role of amplifying cultural stock in each period by using existing stock and their effort as the control variable. In addition, there is no-human capital and no-uncertainty; that is, individuals can precisely predict the amount of cumulative culture even though they cannot memorize what they have learned.

Specifically, their reproduction is according to the following Cobb-Dougras function[iii],

$$Y_t = Y(K_t, h_t) = K_t^{\alpha} h_t \tag{1}$$

where Y_t is the reproduced culture at time t, K_t is cultural stock, and h_t is the amount of effort allocated for reproduction. While it is called capital share in economics, we here define α as the parameter of cultural quality: how existing culture can contribute to reproduction. This is an analogy of the case that academic papers are often evaluated by the number of citations. Therefore, the function intuitively means the process by which individuals make new culture by mixing existing culture and their effort just like researchers write new papers by referring to previous studies.

It is important to note that we do not impose any constraints on α. In addition to ordinary increasing function, for the model deals with culture, Y_t can be a decreasing function with respect to K_t if we assume the easiest culture is likely to be made first and cultural accumulation gradually lessens the room for future reproduction[iv][v]. Therefore, we consider α to be both positive and negative. This assumption allows for richer transitional dynamics which we shall confirm later.

Then, only the fraction of Y_t is assumed to be evaluated and inherited to the future as flow[vi],

$$M_t = M(K_t, h_t) = pY_t = pK_t^{\alpha} h_t \tag{2}$$

where the amount of cultural flow is represented by M_t, and p is the exogenous variable for its probability, $0 < p < 1$. Although exogenous probability is a strong simplification, it does not affect the main implications of the model.

Finally, we can set the following differential equation for the cultural stock[vii],

$$\dot{K} = M(K_t, h_t) - \delta K_t \tag{3}$$

where δK_t denotes depreciation and $0 < \delta < 1$. In the cumulative culture, obsolescence of previous knowledge or physical depreciation of storage media would be practical examples.

In addition to the dynamics of state variable, objective function needs to be defined for microfoundation. We assume it is composed of two functions; u_1: positive utility through evaluation and u_2: negative utility through acquisition cost of cultural stock. If individuals wish to maximize their utility over an infinite horizon, therefore, objective function can be set as follows:

$$U = \int_{t=0}^{\infty} e^{-\rho t}[u_1(M(K_t, h_t)) - u_2(K_t)]dt \tag{4}$$

where ρ is time preference and $\rho > 0$. Specifically, let u_1 and u_2 be CRRA and linear, respectively.

$$u_1 = \frac{M_t^{1-\frac{1}{\sigma}} - 1}{1 - \frac{1}{\sigma}},\ u_2 = \eta K_t \tag{5}$$

σ is the elasticity of intertemporal substitution and $\sigma > 0$, η is the acquisition cost per a unit of culture and $\eta > 0$. Since the curvature of u_1 increases as σ approaches zero, we can also interpret σ as the parameter of creativity: how much incentive do individuals have for cultural reproduction[viii].

That is all of the assumptions. Individuals in the model reproduce new culture by using existing cultural stock and their effort for the utility stemming from its evaluation. However, on the other hand, they have to decide the optimal amount of effort due to the acquisition cost which increases proportionately to cultural stock. Particularly in a model with no-human capital, dynamic optimization problem clearly appears since current evaluation leads to increasing future acquisition cost. Thus, we can say that their learning schedule is based on a preference for "evaluated smoothing" under the constraint of acquisition cost.

3. Steady State

According to the above settings, the dynamic optimization problem is given by [ix],

$$\begin{aligned} &\max_{h_t} U = \int_{t=0}^{\infty} e^{-\rho t}[u_1(M(K_t, h_t)) - u_2(K_t)]dt \\ &\text{s.t. } \dot{K} = M(K_t, h_t) - \delta K_t \\ &\quad K_0 > 0 : given \end{aligned} \tag{6}$$

The Hamiltonian expression can be written as,

$$J = e^{-\rho t}[u_1(M_t) - u_2(K_t)] + v_t(M_t - \delta K_t) \tag{7}$$

where v_t denotes costate variable associated with $\dot{K}$. By solving the problem with substituting for (3)[x], we can obtain the Euler equation[xi].

$$\frac{\dot{M}}{M} = \sigma\left(\eta M_t^{\frac{1}{\sigma}} - \delta - \rho\right) \tag{8}$$

Then, the following equations are also derived which determine the transitional dynamics of the model by assuming $\dot{K} = 0$ in (3) and $\dot{M}/M = 0$ in (8) and using (2),

$$h^* = \frac{\delta}{p}K^{*1-\alpha} \tag{9}$$

$$K^* = \left[\left(\frac{\rho+\delta}{\eta}\right)^{\sigma}\frac{1}{ph^*}\right]^{\frac{1}{\alpha}} \tag{10}$$

where (9) and (10) denote $\dot{K} = 0$ and $\dot{h} = 0$ loci, respectively. Finally, we get the steady-state values of control and state variables from the above simultaneous equations.

$$h^* = \frac{\delta}{p}\left[\left(\frac{\rho+\delta}{\eta}\right)^{\sigma}\frac{1}{\delta}\right]^{1-\alpha},\ K^* = \left(\frac{\rho+\delta}{\eta}\right)^{\sigma}\frac{1}{\delta} \tag{11}$$

Three explicit properties can be found in K*: the steady-state amount of cultural stock[xii].

First and most importantly, α and p are not included here. This means, in the steady-state, the amount of cultural stock would not change even if we controlled its quality and the probability of evaluation.

Second, for the other included parameters, $K^*_\sigma > 0$, $K^*_\rho > 0$, and $K^*_\eta < 0$ hold, respectively. It would be natural that high creativity and low acquisition cost provide more cultural stock. In contrast, we have to take account of the possibility for the effect of time preference to be less in reality. Impatience in the model certainly increases K^* since it makes individuals care relatively little about future acquisition cost; however, this should be weakened if individuals can memorize and reuse what they have learned without incurring additional costs. In other words, the effect is highly attributed to the simplication of the model: no-human capital.

Third, δ works both positively and negatively on K^*. Intuitively, even the high depreciation rate directly decreases cultural stock, it can also contribute to the increase indirectly by reducing future acquisition cost which results in stimulating reproduction. This is the reason why only the positive effect is influenced by σ. We can then confirm the following specific condition by calculating under which the positive effect exceeds the negative effect.

$$\sigma > \frac{\rho+\delta}{\delta} \tag{12}$$

Therefore, contrary to our intuition, high depreciation rate could increase K^* if individuals were enough creative to satisfy the condition[xiii].

4. Dynamics

Although we have dealt thus far with K^* and its properties, they are not meaningful until the stability of each transitional dynamics is confirmed. As shown in (9) and (10), the form of loci varies depending on the exogenous parameter. Specifically, it differs according to the following range: $\alpha < -1$, $-1 < \alpha < 0$, $0 < \alpha < 1$, and $1 < \alpha$.

Case1: $0 < \alpha < 1$

If we assume $0 < \alpha < 1$, that is, diminishing returns to existing cultural stock, transitional dynamics for the model takes the oscillation path as depicted in

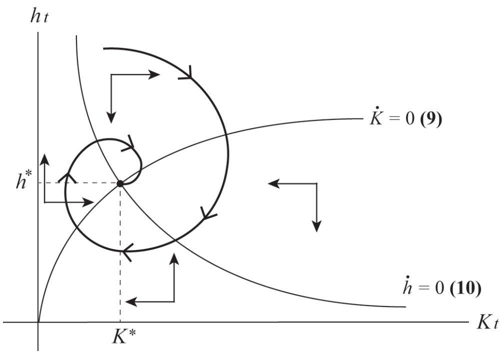

Figure 1 Phase diagram in $0 < \alpha < 1$

Intuitively, this oscillation implements the following iteration;

1. K^* starts to accumulate by reproduction.
2. Optimal K^* gradually decreases by the increase of acquisition cost.
3. K^* finally declines since negative flow surpasses positive flow.
4. Optimal K^* increases again by the decrease of acquisition cost, and back to 1.

Therefore, on the assumption of diminishing returns, cultural stock eventually converges to the steady state while repeating boom and recession.

Case2: $1 < \alpha$

On the other hand, in the case of increasing returns to existing cultural stock, our model takes the dynamics with saddle-path stability shown in figure 2[xiv]. Hence, under this case, there is a slight possibility for the steady-state to be unstable[xv].

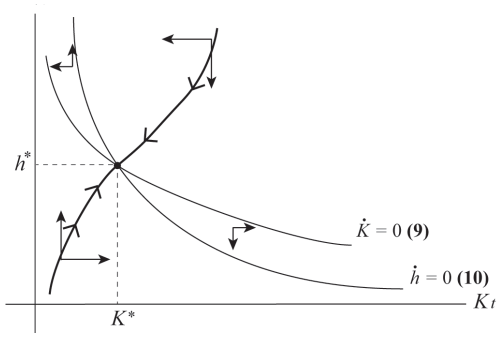

Figure 2 Phase diagram in $1 < \alpha$

Table 1 Growth paths and their stability

	$\alpha < 1$	$-1 < \alpha < 0$	$0 < \alpha < 1$	$1 < \alpha$
Path	stable	stable	stable	saddle
Stability	stable	stable	stable	almost stable

Case3: $-1 < \alpha < 0$

Then, if we let cultural reproduction be a decreasing function of existing cultural stock and α be $-1 < \alpha < 0$, transitional dynamics follows the stable paths depicted in figure 3. Thus, the steady-state is stable regardless of initial values.

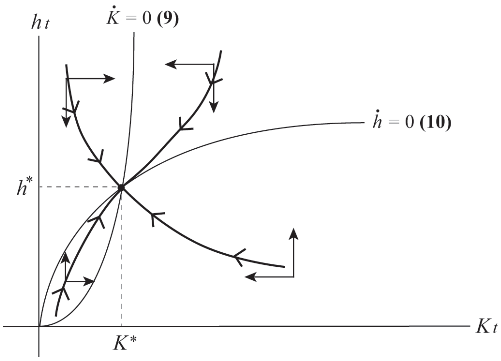

Figure 3 Phase diagram in $-1 < \alpha < 0$

Case4: $\alpha < -1$

Finally, even if α is less than negative one, stability is the same with Case 3 though equation (10) switches to convex as represented in figure 4.

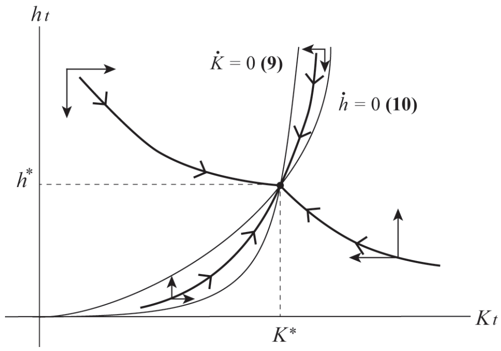

Figure 4 Phase diagram in $\alpha < -1$

Table 1 summarizes all results derived above. Although each transitional dynamic takes a different path, they are all stable except a certain situation in Case 2. Therefore, we conclude that the steady-state is almost stable.

5. Conclusion

As a result of the dynamic analysis, the following two main conclusions were obtained:

- The steady-state value of cultural stock is not affected by its quality and evaluation.
- The steady-state is stable except a certain case with increasing returns.

Hence, quality and evaluation are actually neutral to the amount of cultural stock which finally accumulates. This implies, practically, that indirect policies would be more effective to cultural stock by supporting an environment where individuals can easily utilize well-archived culture and thereby exert their creativity, rather than direct policies which interfere in the quality and evaluation of contents themselves. We could consider digital archiving as an example of the former, and awarding or certification system as that of the latter.

This reseach predicts some elemental dynamics of cumulative culture resulting from rationality and expectation and suggests some essentially effective factors to the amount of cultural stock in the long run. Further improvements can be considered in both theoretical and empirical fields. Needless to say, theoretical extentions would make our model more realistic by loosening the aformentioned strong simplifications: homogeneous culture, no-human capital and no-uncertainty, and empirical data would also make our model more persuasive by supporting the existence of concrete cultural traits suitable for proposed dynamics. Despite those limitations, however, our model could work as a benchmark in tackling more complex issues on cumulative culture since its structure and implications are sufficiently generalized and robust.

Appendix

Derivation of the Euler Equation

For optimization, the Hamiltonian must satisfy the following first-order conditions,

$$\frac{\partial J}{\partial h_t} = e^{-\rho t}\frac{\partial u_1}{\partial M_t}\frac{\partial M_t}{\partial h_t} + v_t\frac{\partial M_t}{\partial h_t} = 0 \tag{A-1}$$

$$\frac{\partial J}{\partial K_t} = e^{-\rho t}\left(\frac{\partial u_1}{\partial M_t}\frac{\partial M_t}{\partial K_t} - \frac{\partial u_2}{\partial K_t}\right) + v_t\left(\frac{\partial M_t}{\partial K_t} - \delta\right) = -\dot{v} \tag{A-2}$$

Derivatives associated with objective function can be both obtained from (3).

$$\frac{\partial u_1}{\partial M_t} = M_t^{-\frac{1}{\sigma}} \tag{A-3}$$

$$\frac{\partial u_2}{\partial M_t} = \eta \tag{A-4}$$

By using (A-3), (A-1) is simplied as follows.

$$e^{-\rho t}M_t^{-\frac{1}{\sigma}} = -v_t \tag{A-5}$$

Then, taking logarithms and time derivatives of (A-5) leads to the negative growth rate of costate variable.

$$\rho + \frac{1}{\sigma}\frac{\dot{M}}{M} = -\frac{\dot{v}}{v} \tag{A-6}$$

In terms of (A-2), the same rate can also be derived by dividing both sides by v and substituting (A-3), (A-4) and (A-5).

$$\eta M_t^{-\frac{1}{\sigma}} - \delta = -\frac{\dot{v}}{v} \tag{A-7}$$

Finally, we obtain the Euler equation as (8) from (A-6) and (A-7).

Optimal intellectual property rights

Our model has an additional implication for intellectual property rights if we assume their excludability increases both creativity and acquisition cost per a unit of culture[xvi]. We shall find it convenient here to set σ^{-1} as θ and θ decreases as the excludability gets strong (That is, curvature of u_1 approaches linear). Thus, if we focus on K^*, it shifts to

$$K_{ip}^* = \left(\frac{\rho+\delta}{\eta+\eta_{ip}}\right)^{\frac{1}{\theta-\theta_{ip}}} \frac{1}{\delta} \quad \text{(A-8)}$$

where K_{ip}^* is the steady-state amount of cultural stock with intellectual property rights. Accordingly, $K^* < K_{ip}^*$ requires the following condition.

$$\left(\frac{\rho+\delta}{\eta}\right)^{\frac{1}{\theta}} < \left(\frac{\rho+\delta}{\eta+\eta_{ip}}\right)^{\frac{1}{\theta-\theta_{ip}}} \quad \text{(A-9)}$$

By rearranging and taking logarithms, approximately we get,

$$\frac{\eta_{ip}/\eta}{\theta_{ip}/\theta} < \ln\left(\frac{\rho+\delta}{\eta}\right) \quad \text{(A-10)}$$

(A-10) is the condition for intellectual property rights to be effective in the long run, where θ_{ip}/θ is the increasing rate of creativity and η_{ip}/η is that of acquisition cost; each denotes positive and negative contribution[xvii]. Therefore, we can say that the rights are useful only if positive per negative contribution is less than the composition of exogenous parameters: $\ln\left(\frac{\rho+\delta}{\eta}\right)$

Notes

i) See Mesoudi(2011b), p.21. and pp.177-188., for example.

ii) Specifically, this means the sum of all information stored in goods or individuals which corresponds to the term "genotype" in biology. Because of the difficulty of its quantification, however, most empirical studies analyze cultural "phenotype" which is the observable characteristics as a result of background information such as shape, color and motion.

iii) Linearity in h_t is just for the simplification. We can derive the same main conclusions by using more generalized forms including Cobb-Douglas and even CES production function.

iv) Romer(2011), pp.103-104. assumes the same condition in the field of R&D.

v) Hence, $|\alpha|$ would be more accurate for the parameter of quality rather than α.

vi) Csikszentmihalyi(2014) proposes similar framework form a viewpoint of creativity named *systems model*.

vii) Equation (3) has non-constant solutions for any $\alpha \neq 0$.

viii) Concretely, u_1 becomes logarithmic when σ equals one and approaches linear as σ increases.

ix) The transversality condition is not required here since individuals get utility from the evaluation which corresponds not to consumption but to savings in macroeconomic models.

x) See in Appendix.

xi) Note that this equation does not explicitly indicate individuals' dynamic decision-making because, unlike general macroeconomic models, they obtain utility not directly from its control variable but indirectly from evaluated culture.

xii) Notice that, even while the steady-state, contents still continue to change because K^* just denotes the equivalence between M_t and δK_t.

xiii) Since the right-hand side of the condition must be greater than one, $K_\delta^* < 0$ holds if we assume u_1 to be logarithmic.

xiv) We can verify the slope of each loci in the neighbor of the steady-state by log-linearization. If we set $\hat{h}_t = (h_t - h^*)/h^*$ and $\hat{K}_t = (K_t - K^*)/K^*$, equation (9) and (10) can be linearized as $\hat{h}_t = (1-\alpha)\hat{K}_t$ and $\hat{h}_t = -\alpha\hat{K}_t$, respectively. Thus, (10) has the steeper slope in the case $1 < \alpha$.

xv) In addition, we shall find that unstable regions are getting smaller as α increases by the ratio between the slopes of both linearized loci: $\lim_{\alpha\to\infty}(1-\alpha)/-\alpha = 1$

xvi) Note that this analysis is just from the viewpoint of social planner. The one who gets more incentive and the one who owes additional acquisition cost would be diffrent in reality.

xvii) If we adhere to using σ, (A-9) becomes $\frac{\eta_{ip}/\eta}{(\sigma+\sigma_{ip})/\sigma_{ip}} < \ln\left(\frac{\rho+\delta}{\eta}\right)$.

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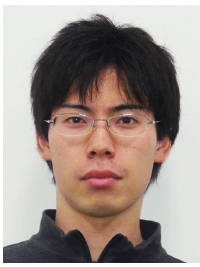

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